Sample Lab: Leslie Model with Logistic Dose Response Model

Background

a. Leslie Model

Leslie matrix (model) is a discrete, non-negative, age-structured model, which describes the development, mortality, and reproduction of organisms over a period of time. As one of the most popular models in population ecology, it was invented by Patrick H. Leslie in 1945. Thus a Leslie matrix contains: age-specific fertilities in the first row, age-specific survival probabilities in the subdiagonal, and zeros elsewhere.

A typical Leslie model is written as:

$$N_{t+1} = LN_t \qquad \text{Eq.1}$$

which can be expanded into:

$$\begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_k \end{bmatrix}_{t+1} = \begin{bmatrix} F_0 & F_1 & \cdots & F_k \\ S_0 & 0 & \cdots & 0 \\ 0 & \ddots & \cdots & 0 \\ 0 & 0 & S_{k-1} & 0 \end{bmatrix} \begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_k \end{bmatrix}_t \qquad \text{Eq.2}$$

where,

$n_{k,t+1}$=number of organisms in age class k at time t+1

F_k=fertility, the per capita average number of female offspring born from mother of age class k

S_k=the fraction of individuals that survives from age class t to t+1

b. Leslie Model coupled with Dose Response Model

Traditionally, ecological risk assessment of pesticides is based on the risk, between measured environment concentrations and laboratory tested concentrations which cause biological effects. This approach has two drawbacks:

1. difficult to relate risk ratios to ecological values (e.g., population size)
2. safety factors used in estimating risk ratios contain great uncertainty which easily cause over or under protection

As a solution, adding population models to the ecological risk assessment process can bridge the gap between measurements and protection goals in the following aspects:

1. reduce uncertainty in extrapolation individual level to population level
2. identify worse scenarios with less effort
3. integrate the effects of multiple stresses
4. reduce the use of laboratory animals

When taking account the reduction of fertility due to the increased population size, the fertility is modified into:

$$F'_x = F_x e^{-\gamma N} \qquad \text{Eq.3}$$

Where,

F'_x is the adjusted fertility rate of age class x

F_x is the maximum fertility rate of age class x

γ is the fertility decay coefficient

N is total population size

When considering the impacts from chemical exposure the survival rate is modified into:

$$S_{tox,x}=S_x(1-M_{48}) \qquad \text{Eq.4}$$

where,

$S_{tox,x}$ is survival rate after toxic for age class x

S_x is the maximum survival rate of age class x

M48 is the 48 hour mortality rate based on logistic dose response model which equals:

$$M_{48} = \frac{1}{1 + e^{-\alpha InC_0 (\frac{1}{2})^{\frac{t}{HL}} - \beta}} \qquad \text{Eq.5}$$

where,

α, β are coefficients of logistic dose response function

C_0 is chemical initial concentration

HL is chemical half life

Computer Lab Procedure

1. Visit http://www.ubertool.org/lesliedr_input.html
2. On the input page, choose '4' as the number of age classes, you will then see the following matrix:

$$L = \begin{bmatrix} F_1 & F_2 & F_3 & F_4 \\ S_1 & 0 & 0 & 0 \\ 0 & S_2 & 0 & 0 \\ 0 & 0 & S_3 & 0 \end{bmatrix}$$

   The f values represent fecundity measures for each of the 4 age classes (the expected number of offspring for each individual). The s values represent the probability of surviving from one age class to the next. For example, if s1=0.8 then a newborn (age=0) has an 80% chance of surviving to be age=1. Enter values in the Leslie matrix for fecundity (0,1,1.5,1.2) and survival (0.8,0.5,0.25). Your matrix should look like this:

$$L = \begin{bmatrix} 0 & 1 & 1.5 & 1.2 \\ 0.8 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.25 & 0 \end{bmatrix}$$

3. Enter values in the initial population vector, this represents the starting population values of each of the 4 age classes at the start of the simulation:

$$N_0 = \begin{bmatrix} 45 \\ 18 \\ 11 \\ 4 \end{bmatrix}$$

4. Use default values for other inputs
5. Click 'submit' button, which will direct you to the output page that show the results of the Leslie matrix model over time.
6. On the output page, save the matrix 'Individuals Over Time' and the figure for future comparison.
7. Go to the input page and keep everything the same as above except change the value for 'Chemical half life (days)' to '0.6.' Resubmit the model and save the results.
8. Go to the input page a third time to run the model. Adjust the value for 'Intensity of the density dependence (γ)' from '0.00548' to '0.6', '4', and '0' separately. Resubmit the model and save the results.

Lab Questions

1. What are the assumptions of Leslie model you have built?
2. Examine your figure. Is the population increasing, stable, or declining?
3. Go to the input page and change the value for 'Chemical half life (days)' to '0.6':
    a. How does a chemical's half life impact the population?
    b. Which age class is most sensitive to this change?
4. Adjust the value for 'Intensity of the density dependence (γ)' from '0.00548' to '0.6', '4', and '0' separately. Compare among different output figures, what is the relationship between parameter 'γ' and the 'shape' of population size over time?
5. How do parameters 'α' and 'β' of logistic dose-response model impact population size (hint: look at the fertility rate)?